

$0\nu 2\beta$ NUCLEAR MATRIX ELEMENTS AND NEUTRINO MAGNETIC MOMENTS

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We compare different methods of obtaining the neutrinoless double beta decay nuclear matrix elements (NME). On the example of ^{76}Ge we use the NME to calculate the Majorana neutrino transition magnetic moments, generated through particle-particle R -parity violating loop diagrams within the minimal supersymmetric standard model.

1. Introduction

The neutrinoless double beta ($0\nu 2\beta$) decay is a hypothetical nuclear process, in which two simultaneous beta decays occur in one nucleus, whereas some exotic mechanism prevents the neutrinos to be emitted. It would result of course in the lepton number violation ($\Delta L = 2$) and therefore it is forbidden by the standard theory. However, in certain exotic models of physics beyond the standard model, such processes are allowed and sought in a number of experiments. The observation of a $0\nu 2\beta$ decay will open a completely new area of physics, which by far is only a speculation.

The theory of this exotic decay involves two parts: firstly, as this is a nuclear process it requires a careful calculation of the nuclear matrix elements; secondly, an exotic mechanism must be proposed, which would suppress the emission of the neutrinos. The $0\nu 2\beta$ half-life may therefore be expressed in a factorized form as

$$(T^{0\nu})^{-1} = G^{0\nu} |M^{0\nu}|^2 |\langle m_{ee} \rangle|^2, \quad (1)$$

where $G^{0\nu}$ is the exactly calculable phase-space factor, $M^{0\nu}$ is the NME, and $\langle m_{ee} \rangle$ is the so-called effective neutrino mass, which represents the non-standard interactions involved. We are not going to discuss the latter part here.

In this communication, we are going to compare different methods of obtaining the nuclear matrix elements (NME) for the $0\nu 2\beta$ decay. As an application, we will use them to calculate the Majorana neutrino transition magnetic moments within the minimal supersymmetric standard model with broken R -parity.

2. Calculations of the NME: random phase approximation

The random phase approximation (RPA) methods in their simplest form base on the assumption, that the ground state is expressed as a BCS vacuum. Then, a transition amplitude between the 0^+ BCS vacuum of an even–even nucleus and excited states of the neighboring odd–odd nucleus is calculated, representing the transition between the mother nucleus and an intermediate one. A similar amplitude represents the transition between the intermediate and the daughter nucleus. These two parts are then summed over possible excited states of the intermediate virtual nucleus, which are expressed as simple harmonic oscillations above its BCS ground state.

Such simplified picture does not give satisfying results and many variation of the original method appear. These involve the use of quasiparticles (QRPA, pnQRPA), renormalized QRPA (RQRPA), selfconsistent RQRPA (SRQRPA) and others. In QRPA a quasiboson approximation is used, in which the commutation relations for fermions are replaced by bosonic ones, and this procedure seems to be working well for small harmonic excitations, but breaks down when one wants to take into account higher excitations as well. In RQRPA the commutators are approximated in a different way, which introduces the new factor as a normalization constant.

What is more, different schemes of short-range correlations are used, among which the Jastrow and the unitary correlation operator method (UCOM) are most common. Also one may adjust other parameters, such as the nuclear potential (Woods–Saxon, effective mean field etc.), and the strength of the particle-particle (g_{pp}) and particle-hole (g_{ph}) interactions.

All these give a quite big diversity in the outcome of different calculations. At present one may summarize the results of the Jyväskylä and the Tübingen groups,¹ for ^{76}Ge as $3.33 < M^{0\nu} < 6.64$.

3. Calculations of the NME: nuclear shell model

The nuclear shell model (NSM) and its recently announced large-scale version (LSSM) is meant in principle to give exact results. The approach is straightforward and involves defining a valence space, deriving an effective interaction from the Hamiltonian, building, and then finally diagonalizing the Hamiltonian matrix. The first and most obvious obstacle here is, however, the dimension of the matrix in question, which is proportional to

$$\dim \sim \binom{d_\pi}{p} \cdot \binom{d_\nu}{n} \quad (2)$$

where d_π (d_ν) is the dimension of the proton (neutron) subshell, and p (n) is the number of valence protons (neutrons). This feature makes it practically impossible to include in the valence space all important single particle orbits; eg. the spin-orbit partners are usually neglected. Also in the actual calculations the many-body problem is reduced to two-body problem within some mean-field approach.

Table 1. Neutrinoless double beta decay nuclear matrix element $M^{0\nu}$ for ^{76}Ge calculated using three different approaches. The last column shows the corresponding upper limit on the effective neutrino mass (see text for details).

method	$M^{0\nu}$: ranges	central value	$ \langle m_{ee} \rangle \leq$
(R)QRPA	3.33 – 6.64	4.985 ± 1.655	0.22 – 0.43 eV
LSSM	2.22 – 2.81	2.515 ± 0.295	0.51 – 0.65 eV
IBM	4.64 – 5.46	5.050 ± 0.414	0.26 – 0.31 eV

The most advanced large scale shell model calculations for the ^{76}Ge nucleus yield at present $M^{0\nu} = 2.22$ (without higher order contributions to the nuclear current this value increases to 2.58). A little bit older calculations which used the UCOM short-range correlations gave $M^{0\nu} = 2.81$.²

4. Calculations of the NME: interacting boson model

The interacting boson model (IBM) has not been used for this type of calculations before, but it turned out to give results which surprisingly well agree with the (R)QRPA calculations. In this method, a fermionic two-body matrix element is used to obtain the fermions transition operator in second quantization. The matrix elements of this operator are then evaluated in the general seniority scheme. At the end, the fermionic operator is mapped into bosonic transition operator, whose matrix elements are evaluated using realistic IBM wave function.

This method, introduced by Barea and Iachello in Ref.³, yields for the ^{76}Ge nucleus $4.64 < M^{0\nu} < 5.46$, which is in very good agreement with the central values obtained within the (R)QRPA method.

We summarize the various results for the ^{76}Ge nucleus in Tab.1. In the last column we have calculated the effective neutrino mass using the result of the Heidelberg–Moscow experiment, $T_{1/2}^{0\nu}(^{76}\text{Ge}) \leq 1.9 \times 10^{25} \text{ y}$,⁴ and a common phase-space factor $G^{0\nu} = 2.55 \times 10^{-26} \text{ y eV}^2$.

5. Majorana neutrino transition magnetic moments in R -parity violating supersymmetry

The Majorana magnetic moment acts between ν_{iL} and ν_{jL}^c chiral components of Majorana neutrinos, assuming a standard gauge theory with only left-handed neutrinos. As a consequence, it violates the total lepton number by two units $\Delta L = 2$. The effective Hamiltonian H_{eff} of this interaction takes the form

$$H_{\text{eff}} = \frac{1}{2} \mu_{ij} \bar{\nu}_{iL} \sigma^{\alpha\beta} \nu_{jL}^c F_{\beta\alpha} + \text{h.c.}, \quad (3)$$

where F is the electromagnetic field strength tensor, and $\sigma^{\alpha\beta}$ is defined by means of the Pauli matrices, $\sigma^{\alpha\beta} = \frac{1}{2}[\sigma^\alpha, \sigma^\beta]$. We work in the minimal supersymmetric standard model with broken R -parity.⁵ In this setting it is possible to consider

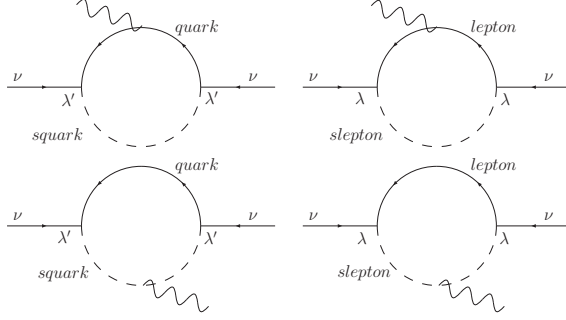
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Fig. 1. Processes leading to Majorana neutrino transition magnetic moments.

processes, in which the Majorana neutrino, despite being electrically neutral, may effectively interact with an external photon. The amplitude of such processes is governed by the magnetic moments of the neutrino. We consider interaction between two Majorana neutrinos, in which the effective interaction vertex is expanded into a particle–particle loop, as depicted on Fig.1

The quark–squark loop contribution, including possible d -quark mixing (V is the CKM matrix), is given by the following formula:⁶

$$\mu_{\nu_{ii'}}^q = (1 - \delta_{ii'}) \frac{12Q_d m_e}{16\pi^2} \sum_{jkl} \left\{ \lambda'_{ijk} \lambda'_{i'kl} \sum_a V_{ja} V_{la} \frac{w_{ak}^q}{m_{da}} - (k \leftrightarrow j) \right\} \mu_B, \quad (4)$$

where w is the loop integral, which arises from the integration over virtual momenta of particles inside the loop,

$$w_{jk}^q = \frac{\sin 2\theta^k}{2} \left(\frac{x_2^{jk} \ln x_2^{jk} - x_2^{jk} + 1}{(1 - x_2^{jk})^2} - (x_2 \rightarrow x_1) \right), \quad (5)$$

and we have denoted the ratio of the quark and squark masses squared by $x_i^{jk} = m_{d_j}^2/m_{d_i}^2$. Here θ^k is the k -th squark mixing angle. The contribution from the slepton–lepton loop does not contain summation over three quark colors, and we do not include the very weak mixing in the charged lepton sector. The formula reads therefore⁶

$$\mu_{\nu_{ii'}}^\ell = (1 - \delta_{ii'}) \frac{4Q_e m_e}{16\pi^2} \sum_{jk} \lambda_{ijk} \lambda_{i'kj} \left(\frac{w_{jk}^\ell}{m_{ej}} - \frac{w_{kj}^\ell}{m_{ek}} \right) \mu_B, \quad (6)$$

where the loop integral is identical as previously with the exception that $x_i^{jk} \rightarrow y_i^{jk} = m_{\ell_j}^2/m_{\ell_i}^2$, and $\theta^k \rightarrow \phi^k$, the slepton mixing angle.

In order to get numerical results, we constrain the model by imposing grand unification at very high energy scale, $M_{\text{GUT}} \approx 1.2 \times 10^{16}$ GeV, thus starting from few free parameters only. These are the common mass of scalars m_0 , the common mass of fermions $m_{1/2}$, common Yukawa coupling constants unification factor A_0 , the ratio of the Higgs vacuum expectation values $\tan \beta$, and the sign of the bilinear up- and down-type Higgs coupling $\text{sgn}(\mu)$. Next, the renormalization group equations are

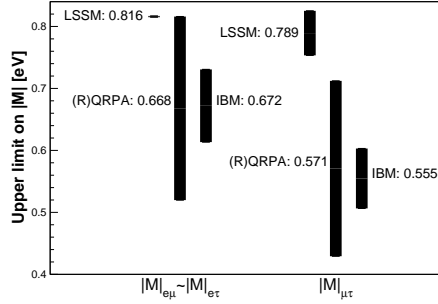


Fig. 2. Ranges for the maximal values of the off-diagonal elements in the neutrino mass matrix, coming from different $0\nu 2\beta$ nuclear matrix elements.

Table 2. Majorana neutrino transition magnetic moments $\mu_{\nu ij}$ in μ_B for GUT parameters: $A_0 = 100$ GeV, $m_0 = m_{1/2} = 150$ GeV, $\tan \beta = 19$, $\mu > 0$. Ranges correspond to the spread in NME obtained using different methods.

ij	LSSM	(R)QRPA	IBM
lepton-slepton loop mechanism			
$e\mu, e\tau$	$(1.33, 1.33) 10^{-15}$	$(8.46, 13.3) 10^{-16}$	$(9.99, 11.9) 10^{-16}$
$\mu\tau$	$(1.23, 1.35) 10^{-15}$	$(7.02, 11.6) 10^{-16}$	$(8.28, 9.86) 10^{-16}$
quark-squark loop mechanism (without d-quarks mixing)			
$e\mu, e\tau$	$(9.49, 9.49) 10^{-17}$	$(6.04, 9.49) 10^{-17}$	$(7.13, 8.50) 10^{-17}$
$\mu\tau$	$(8.69, 9.52) 10^{-17}$	$(4.95, 8.22) 10^{-17}$	$(5.84, 6.96) 10^{-17}$
quark-squark loop mechanism (with d-quarks mixing)			
$e\mu, e\tau$	$(8.22, 8.22) 10^{-17}$	$(5.24, 8.22) 10^{-17}$	$(6.18, 7.36) 10^{-17}$
$\mu\tau$	$(7.24, 7.93) 10^{-17}$	$(4.13, 6.85) 10^{-17}$	$(4.87, 5.80) 10^{-17}$

used to derive the values of the couplings and mass parameters at low energy scales. The R -parity violating trilinear couplings λ and λ' , which cannot be derived from the GUT constraints, are assessed from the experimental neutrino mass matrix. This procedure allows to calculate the transition magnetic moments (see Ref.⁶ for details).

Fig.2 presents how the off-diagonal elements in the neutrino mass matrix depend on the nuclear matrix element used. Each bar shows the range for a given method, with the number being its central value. One sees immediately, that the (R)QRPA and IBM methods agree very well with each other, while the shell model calculations give substantially higher results. This trend is of course preserved also in the magnetic moment data, which are shown in Tabs.2 and 3. We have included calculations for two cases, one with ‘small’ GUT scale unification parameters, the other with ‘large’ ones. For each case three possibilities were considered, namely the lepton–slepton loop, and the quark–squark loop with or without additional d -quark mixing. One sees that the differences in the magnetic moments for various nuclear matrix elements are negligible, and that the ranges often overlap with each other. As expected, the LSSM gives the highest results, but the difference between

Table 3. Majorana neutrino transition magnetic moments $\mu_{\nu_{ij}}$ in μ_B for GUT parameters: $A_0 = 500$ GeV, $m_0 = m_{1/2} = 1000$ GeV, $\tan \beta = 19$, $\mu > 0$. Ranges correspond to the spread in NME obtained using different methods.

ij	LSSM	(R)QRPA	IBM
lepton-slepton loop mechanism			
$e\mu, e\tau$	(3.48, 3.48) 10^{-17}	(2.22, 3.48) 10^{-17}	(2.62, 3.12) 10^{-17}
$\mu\tau$	(3.26, 3.57) 10^{-17}	(1.86, 3.08) 10^{-17}	(2.19, 2.61) 10^{-17}
quark-squark loop mechanism (without d-quarks mixing)			
$e\mu, e\tau$	(2.53, 2.53) 10^{-18}	(1.61, 2.53) 10^{-18}	(1.90, 2.27) 10^{-18}
$\mu\tau$	(2.32, 2.54) 10^{-18}	(1.32, 2.19) 10^{-18}	(1.56, 1.86) 10^{-18}
quark-squark loop mechanism (with d-quarks mixing)			
$e\mu, e\tau$	(2.15, 2.15) 10^{-18}	(1.37, 2.15) 10^{-18}	(1.62, 1.93) 10^{-18}
$\mu\tau$	(2.03, 2.22) 10^{-18}	(1.16, 1.92) 10^{-18}	(1.36, 1.62) 10^{-18}

for example the upper (R)QRPA value and the lower LSSM one is roughly of the order of 10%. Of course, the predictions on the half-life of the $0\nu 2\beta$ decay, or the effective neutrino mass may differ in a much more serious manner. In such a case the (R)QRPA and IBM methods are worth recommendation.

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